Prediction of Turbulent Heat Transfer Downstream of an Abrupt Pipe Expansion

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Turbulent flow and heat transfer characteristics downstream of a sudden circular pipe expansion are predicted by using the full Reynolds stress model. The uniform wall temperature condition is imposed to the downstream wall. The governing differential equations are discretized by finite volume method with power-law scheme. The results show that the precise inlet conditions taken from the experimental data improve the results, but that the overall magnitude of Nusselt number is still under-predicted due to the use of conventional wall functions.

Key Words: Numerical Calculation, Turbulent Heat Transfer, Recirculating Flow, Reynolds Stress Model, k-ε Model, Finite Volume Method

Nomenclature

c_p : Constant pressure specific heat, J/kg°C
h : Step height
k : Turbulent kinetic energy
Nu : Nusselt number
p : Pressure fluctuation
P : Mean pressure or production rate of $k(P = P_{ii})$
Pr : Prandtl number
q''_w : Wall heat flux, J/m^{2}s
R : Radius of a large r pipe
T : Mean temperature
U, V : Mean velocities in x and r directions, respectively
u, v, w : Velocity fluctuations in x, r, and circumferential directions, respectively
$\overline{u_i u_j}$: Reynolds stress
$\overline{u_i \theta}$: Turbulent thermal flux
x, r : Axial and radial coordinates, respectively
y : Distance from a wall
α : Thermal diffusivity, m^2/s
δ_{ij} : Kronecker delta
ε : Dissipation rate of k
θ : Temperature fluctuation
μ, ν : Dynamic and kinematic viscosities, respectively
ρ : Fluid density, kg/m^3
τ_w : Wall shear stress

Subscripts

max : Local maximum
p : Value at the node adjacent to a wall
w : Wall value

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1. Introduction

Recirculating flow due to a sudden change of the flow area is observed in many engineering applications such as heat exchangers, gas turbines, and combustion chambers. The flowfield consists of the separation of a boundary layer, a curved free shear layer with high turbulence levels, a recirculation zone, a reattachment, and a secondary vortex in the corner of step. Accordingly, it is difficult to understand the characteristics of this complex flow. In particular, recirculating flow downstream of a sudden axisymmetric expansion enhance the heat transfer coefficient several times greater than that for fully developed turbulent pipe flow at the same Reynolds number, which is mainly attributed to increase in the turbulent kinetic energy due to flow separation. The purpose of present work is to obtain the numerical predictions for the velocity and temperature fields of the flow mensioned above, and also to compare the performance of Reynolds stress model with that of k-ε model.

Since circular pipe with sudden expansion is a common geometry usually encountered in engineering devices, there has been a lot of investigations on this flow. Most of experiments(Baughn et al., 1984; Zemanick and Dougall, 1970) were carried out under uniform wall heat flux condition. However, recent work done by Baughn et al. (1989) employed uniform wall temperature condition. They measured velocity and temperature fields as well as heat transfer coefficients to provide a data base for the evaluation of turbulence models. As numerical investigations, Gooray et al.(1985) and Amano et al.(1983) reported the preditions with k-ε model. Their evaluations of the model were focused mainly on the prediction of heat transfer coefficients. Recently, Prud'homme and Elghobashi(1986) used a low-Reynolds-number version of Reynolds stress model, which employes algebraic model for thermal fluxes, in the calculation of the same flow. Their results show that the computed flow structure and heat transfer are in good agreement with the experimental data. However, the performance on predicting temperature field was not reported.

This work presents the predictions with full Reynolds stress model on both velocity and temperature fields under uniform wall temperature condition. The results have been compared with those obtained with k-ε model as well as experimental data measured by Baughn et al.(1989)

2. Governing Equations and Turbulence Models

Reynolds-averaged Navier-Stokes equations and energy equation for steady, incompressible, axisymmetric, turbulent flow can be written in cylindrical coordinates, as follows.

$$\rho U\frac{\partial U}{\partial x}+\rho V\frac{\partial U}{\partial r}=\frac{\partial}{\partial x}\left[\mu\frac{\partial U}{\partial x}-\rho\overline{u^2}\right]+\frac{1}{r}\frac{\partial}{\partial r}\left[r\left(\mu\frac{\partial U}{\partial r}-\rho\overline{uv}\right)\right]-\frac{\partial P}{\partial x} \tag{1}$$

$$\rho U\frac{\partial V}{\partial x}+\rho V\frac{\partial V}{\partial r}=\frac{\partial}{\partial x}\left[\mu\frac{\partial V}{\partial x}-\rho\overline{uv}\right]+\frac{1}{r}\frac{\partial}{\partial r}\left[r\left(\mu\frac{\partial V}{\partial r}-\rho\overline{v^2}\right)\right]-\mu\frac{V}{r^2}+\rho\frac{\overline{w^2}}{r}-\frac{\partial P}{\partial r} \tag{2}$$

$$\rho U\frac{\partial T}{\partial x}+\rho V\frac{\partial T}{\partial r}=\frac{\partial}{\partial x}\left[\frac{\mu}{Pr}\frac{\partial T}{\partial x}-\rho\overline{u\theta}\right]+\frac{1}{r}\frac{\partial}{\partial r}\left[r\left(\frac{\mu}{Pr}\frac{\partial T}{\partial r}-\rho\overline{v\theta}\right)\right] \tag{3}$$

To close the above equations for mean velo cities and mean temperature, Reynolds stress model solves transport equations for Reynolds stresses and turbulent thermal fluxes. The transport equation for Reynolds stress tensor is written as

$$\frac{\partial}{\partial x_k}(U_k\overline{u_iu_j})=D_{ij}+P_{ij}+\Phi_{ij}-\varepsilon_{ij} \tag{4}$$

where

$$D_{ij}=-\frac{\partial}{\partial x_k}(\overline{u_iu_ju_k})$$

: turbulent diffusion

$$P_{ij}=-\left[\overline{u_ju_k}\frac{\partial U_i}{\partial x_k}+\overline{u_iu_k}\frac{\partial U_j}{\partial x_k}\right]$$

: production rate

$$\Phi_{ij}=\frac{p}{\rho}\overline{\left[\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right]}$$

: pressure-strain rate interaction

$$\varepsilon_{ij}=2\nu\overline{\frac{\partial u_i}{\partial x_k}\frac{\partial u_j}{\partial x_k}}$$

: dissipation rate

The viscous diffusion term is neglected in Eq. (4), because the empirical wall function is employed in the near-wall region including viscous sublayer.

As a model for turbulent diffusion, Daly and Harlow's simple gradient diffusion model(1970) is used in this work.

$$D_{ij}=\frac{\partial}{\partial x_k}\left[c_s\frac{k}{\varepsilon}\overline{u_ku_l}\frac{\partial\overline{u_iu_j}}{\partial x_l}\right] \tag{5}$$

Based on the assumption of local isotropy of the smallest scale, following isotropic model for dissipation is employed.

$$\varepsilon_{ij}=\frac{2}{3}\varepsilon\delta_{ij} \tag{6}$$

The pressure-strain rate interaction term reflects the return-to-isotropy characteristics of turbulent motion. The Poisson equation for pressure fluctuation indicates that there are two different kinds of interaction; one by turbulent motion($\Phi_{ij,1}$) and the other by mean motion($\Phi_{ij,2}$), and also that there are the effects of solid wall boundary($\Phi_{ij,w}$).

$$\Phi_{ij}=\Phi_{ij,1}+\Phi_{ij,2}+\Phi_{ij,w} \tag{7}$$

For the interaction term by turbulence, following Rotta's proposal(1951) is widely used in second-order closures.

$$\Phi_{ij,1}=-c_1\frac{\varepsilon}{k}(\overline{u_iu_j}-\frac{2}{3}\delta_{ij}k) \tag{8}$$

Launder et al.(1975) proposed a model for the interaction term by mean motion. The simplified version of the model is employed in this work.

$$\Phi_{ij,2}=-c_2(P_{ij}-\frac{2}{3}\delta_{ij}P) \tag{9}$$

Following Gibson and Launder(1978), we adopt the following model for wall reflection term.

$$\Phi_{ij,w}=c_1'\frac{\varepsilon}{k}\left(\overline{u_n^2}\delta_{ij}-\frac{3}{2}\overline{u_nu_i}\delta_{nj}-\frac{3}{2}\overline{u_nu_j}\delta_{ni}\right)f+c_2'\left(\Phi_{nn,2}\delta_{ij}-\frac{3}{2}\Phi_{ni,2}\delta_{nj}-\frac{3}{2}\Phi_{nj,2}\delta_{ni}\right)f \tag{10}$$

where f is a wall damping function defined by

$$f=\frac{c_\mu^{3/4}k^{3/2}}{\kappa}(R-r)\varepsilon \tag{11}$$

The equation for dissipation rate of turbulent kinetic energy used in this work is same as that proposed by Launder et al.(1975) as follows.

$$U_k\frac{\partial\varepsilon}{\partial x_k}=\frac{\partial}{\partial x_k}\left[c_\varepsilon\frac{k}{\varepsilon}\overline{u_ku_l}\frac{\partial\varepsilon}{\partial x_l}\right]+c_{\varepsilon1}\frac{\varepsilon}{k}P-c_{\varepsilon2}\frac{\varepsilon^2}{k} \tag{12}$$

Transport equation for thermal flux can be written by

$$\frac{\partial}{\partial x_k}(U_k\overline{u_i\theta}=D_{i\theta}+P_{i\theta}+\Phi_{i\theta}-\varepsilon_{i\theta} \tag{13}$$

where

$$D_{i\theta}=-\frac{\partial}{\partial x_k}\{\overline{u_iu_k\theta}\}$$

: turbulent diffusion

$$P_{i\theta}=-\left[\overline{u_iu_k}\frac{\partial T}{\partial x_k}+\overline{u_k\theta}\frac{\partial U_i}{\partial x_k}\right]$$

: production rate

$$\Phi_{i\theta}=\overline{\frac{p}{\rho}\left(\frac{\partial\theta}{\partial x_i}\right)}$$

: pressure-temperature gradient interaction

$$\varepsilon_{i\theta}=(\alpha+\nu)\overline{\frac{\partial u_i}{\partial x_k}\frac{\partial\theta}{\partial x_k}}$$: dissipation rate

Daly and Harlow's gradient model(1970) is adopted to the turbulent diffusion term as follows.

$$D_{i\theta}=\frac{\partial}{\partial x_k}\left[c_\theta\frac{k}{\varepsilon}\overline{u_ku_l}\frac{\partial\overline{u_i\theta}}{\partial x_l}\right] \tag{14}$$

As for the dissipation term, it is assumed that turbulence is isotropic for high-Reynolds-number flows and consequently it is zero because there is no isotropic first-order tensor.

To pressure-scrambling term, $\Phi_{i\theta}$, there are three different contributions which are due to turbulent motion($\Phi_{i\theta,1}$), mean motion($\Phi_{i\theta,2}$), and wall reflection($\Phi_{i\theta,w}$).

$$\Phi_{i\theta}=\Phi_{i\theta,1}+\Phi_{i\theta,2}+\Phi_{i\theta,w} \tag{15}$$

As a model for $\Phi_{i\theta,1}$, Monin's popular model(1965) is adopted in this work as follows.

$$\Phi_{i\theta,1}=-c_{1\theta}\frac{\varepsilon}{k}\overline{u_i\theta} \tag{16}$$

The destruction of production model(Launder, 1976) which is used for $\Phi_{i\theta,1}$ is written as

$$\Phi_{i\theta,2}=c_{2\theta}\overline{u_k\theta}\frac{\partial U_i}{\partial x_k} \tag{17}$$

The effects of wall reflection are assumed to be small unlike the case in stress equation. Thus, the

Table 1 Model constants

c_1	c_2	c_1'	c_2'	$c_{\varepsilon1}$	$c_{\varepsilon2}$	$c_{1\theta}$
1.8	0.6	0.5	0.3	1.44	1.92	3.0
$c_{2\theta}$	c_S	c_ε	c_θ	κ	c_μ	Pr
0.5	0.22	0.18	0.11	0.4187	0.09	0.71

Table 2 Inlet conditions

	Condition ①	Condition ②
U	experiment(Baughn, et al., 1989)	experiment(Baughn, et al., 1989)
V	0	0
T	uniform temperature	uniform temperature
k	$0.003U^2$	$\frac{1}{2}(\overline{u^2}+\overline{v^2}+\overline{w^2})$
$\overline{u^2}$, $\overline{v^2}$, $\overline{w^2}$	$\frac{2}{3}k$	experiment(Laufer, 1954)
$\overline{uv}$	$-c_\mu\frac{k^2}{\varepsilon}\frac{\partial U}{\partial r}$	experiment(Laufer, 1954)
$\overline{u\theta}$, $\overline{v\theta}$	0	0
ε	$c_\mu\frac{k^{3/2}}{0.001D}$	$-c_\mu\frac{k^2}{\overline{uv}}\frac{\partial U}{\partial r}$

corresponding term is neglected in this work. However, the following Gibson and Launder's model(1978) for this term is also tested in this work.

$$\Phi_{i\theta,w}=-c'_{1\theta}\frac{\varepsilon}{k}\overline{u_n\theta}\,\delta_{in} \tag{18}$$

where the model constant, $c'_{1\theta}$ is 0.25.

The values of model constants used in this work are listed in Table 1.

The k-ε model used in this work is same as the standard model proposed by Launder and Spalding(1974).

The governing equations are discretized by finite volume method with power-law scheme. And, the solution procedure is based on SIMPLE algorithm(Patankar, 1980).

At the inlet to the computational domain, which coincides with the plane of expansion, the flow is assumed to be fully developed pipe flow. The inlet velocity profile is obtained from experiment(Baughn et al., 1989). Two types of inlet condition are used in this work. And they are listed in Table 2.

At the exit, streamwise gradients of all variables are neglected. The wall functions adopted in the near-wall region are as follows.

$$\frac{U_p}{\left(\frac{\tau}{\rho}\right)_w}c_\mu^{\frac{1}{4}}k_p^{\frac{1}{2}}=\frac{1}{\kappa}ln\left[\frac{Ey_p(c_\mu^{\frac{1}{2}}k_p)^{\frac{1}{2}}}{\nu}\right] \tag{19}$$

$$\frac{(T_w-T_p)}{\dot{q}''_w}\rho c_p c_\mu^{\frac{1}{4}}k_p^{\frac{1}{2}}=\frac{Pr_t}{\kappa}ln\left[\frac{Ey_p(c_\mu^{\frac{1}{2}}k_p)^{\frac{1}{2}}}{\nu}\right]+P_f \tag{20}$$

where E is 9.723 and P_f is obtained from the following equation(Habib and McEligot, 1982).

$$P_f=c_f\left(\frac{Pr}{Pr_t}-1\right)\left(\frac{Pr_t}{Pr}\right)^{\frac{1}{4}} \tag{21}$$

where $C_f=9.24$ and $Pr_t=0.9$

3. Results and Discussion

The present results are obtained by calculating the flow and heat transfer downstream of an abrupt expansion in a circular pipe with Reynolds stress model and k-ε model. The computed results are compared with the experimental data of Baughn et al.(1989), which was obtained under uniform wall temperature condition. Comparisons are made at Reynolds number 17,400 with a small diameter to large diameter ratio of 0.4. The length of the computational domain is sixteen diameter of downstream section.

The results of grid-dependency test for 40×20, 80×30 and 100×30 grid are shown in Fig. 1, where the local Nusselt numbers are normalized

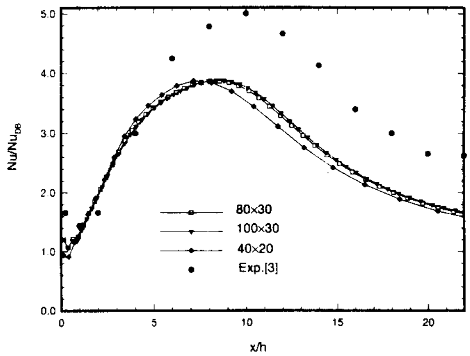


Fig. 1 Effects of grid System
(Nu distributions, Experiment by Baughn et al., 1989)

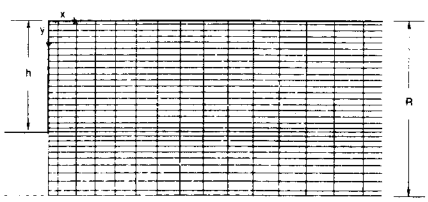


Fig. 2 Grid system(80×30)

by an effective Nusselt number for fully developed pipe flow determined from Dittus-Boelter correlation(Nu_{DB}). Both 80×30 and 100×30 grids produce almost same results. Thus, the grid system of 80×30 shown in Fig. 2 is expected to give grid-independent solutions.

The effects of inlet condition are also tested in this work. Figures. 3(a) to 3(c) show that the solutions are very sensitive to the inlet condition, and that the inlet condition 2 based on the experimental data for turbulent stress components predicts better than the inlet condition 1 in Table 2. Although the magnitudes of Nusselt number are still under-predicted, the location of maximum Nusselt number obtained with inlet condition 2 coincides with the experimental results. Figure 4 shows the effect of wall reflection model of pressure-scrambling term in the thermal flux equation. The use of Gibson and Launder's model(Eq. (18)) dose not improve the results remarkably. Therefore, this term is simply neglected in the following calculations.

Figures 5(a) to 5(c) compare the results of

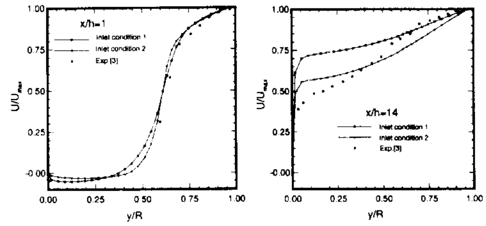


Fig. 3 (a) Effects of inlet condition
(Velocity Profiles, Experiment by Baughn et al., 1989)

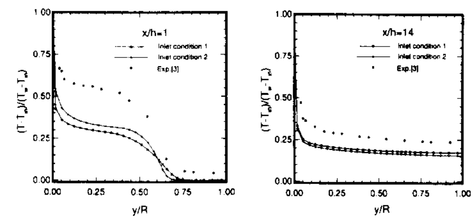


Fig. 3 (b) Effects of inlet condition
(Temperature Profiles, Experiment by Baughn et al., 1989)

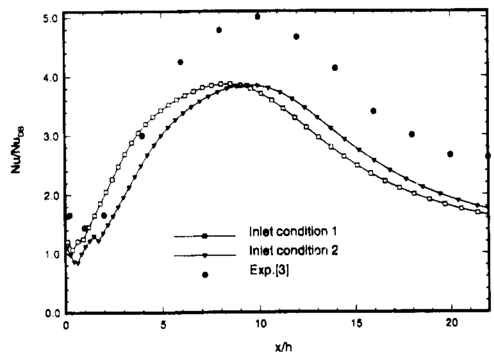


Fig. 3 (c) Effects of inlet condition
(Nu distributions, Experiment by Baughn et al., 1989)

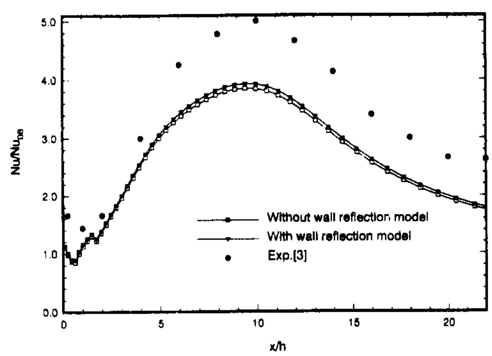


Fig. 4 Effects of wall reflection model for $\Phi_{i\theta}$
(Nu distributions, Experiment by Baughn et al., 1989)

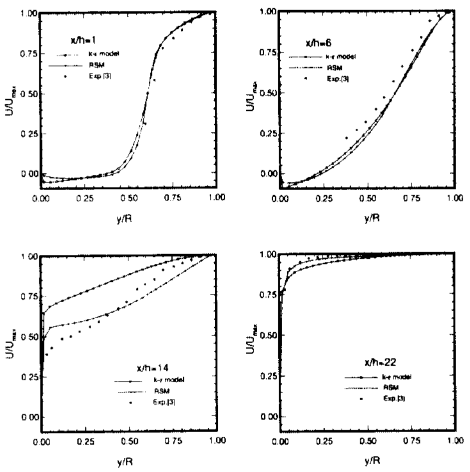


Fig. 5 (a) Predictions with Reynolds stress model and k-ε Model
(Velocity Profiles, Experiment by Baughn et al., 1989)

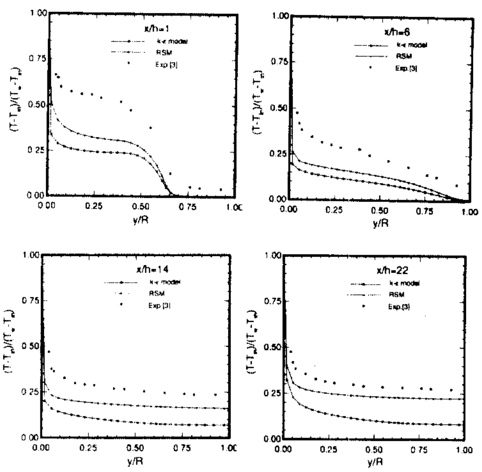
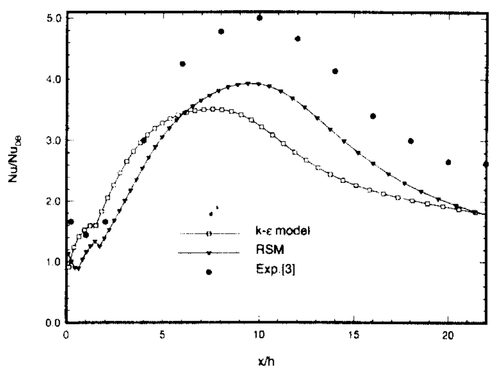


Fig. 5 (b) Predictions with reynolds stress model and k-ε Model
(Temperature Profiles, Experiment by Baughn et al., 1989)



(c) Predictions with reynolds stress model and k-ε Model
(Nu distributions, Experiment by Baughn et al., 1989)

Reynolds stress model with those of k-ε model. In the figures, the Reynolds stress model shows better predictions in all aspects. Especially, the Reynolds stress model predicts the correct locations of maximum and minimum Nusselt numbers. But, the standard k-ε model fails to capture the minimum of Nusselt number caused by the secondary vortex in the corner of the expansion. This seems to be concerned with the fact that the standard k-ε model does not account for the additional effects of streamline curvature in the recirculation zone. While the predicted temperature profiles are qualitatively similar to the expe rimental profiles, quantitative discrepancies, i. e., steeper gradients at the wall are found in the recirculating region, which cause also a discrepancy in the overall magnitude of Nusselt number as shown in Fig. 5(C). The discrepancy seems to result from the conventional log-law wall functions(Eqs. (19) and (20)) employed in this work.

4. Conclusion

The flow and heat trasfer characteristics downstream of a sudden expansion in a circular pipe are predicted by using the full Reynolds stress model. It is found that the precise inlet conditions taken from the experimental data especially for dissipation rate of turbulent kinetic energy remarkably improve the results, but that the wall reflection model of pressure-scrambling term in turbulent thermal flux equation does not affect the results. The results with Reynolds stress model are much better than those with k-ε model in the predictions of velocity and temperature fields as well as the heat transfer coefficients. However, the overall magnitude of Nusselt number is under-predicted due to the use of conventional log-law wall functions.

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